Lab: Kepler's Laws

Purpose: to learn that orbit shapes are ellipses, gravity and orbital velocity are related, and force of gravity and orbital period are related.

Materials: 2 thumbtacks, 1 pencil, string, piece of cardboard

Introduction

Johannes Kepler was a German mathematician, astronomer, and astrologer in the 17th century. He used the astronomer Tycho Brahe's detailed observations of the planets develop a mathematical model to predict the positions on the sky of the planets. He published this model as three "laws" of planetary motion:

> **1st Law: "The orbit of every planet is an ellipse with the Sun at one of the two foci."**
> **2nd Law: "A line joining a planet and the Sun sweeps out equal areas during equal intervals of time."**
> **3rd Law: "The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit."**

Kepler's model was an empirical model, meaning a model that predicts physical events without necessarily explaining what causes them. Some 70 years later Isaac Newton provided a physical explanation for Kepler's laws when he published "*Principia Mathematica*" in 1687. This book described gravity and the laws of motion, the perfect application of which is planets orbiting the Sun in the vacuum of space.

Regarding the words "hypothesis", "theory", and "law": the scientific method goes from hypothesis to theory. While on TV the word "theory" means "hunch", in science "theory" means a well-confirmed explanation of the natural world based on observation and experiment. The word "law" is an antiquated designation and is **not** the final stage of a theory.

To demonstrate orbits, speeds, and periods, students are encouraged to look at the 16-year-time-lapse animation of the stars orbiting Sag A*, the supermassive black hole at the center of the Milky Way. Two groups studied these stars: Genzel (http://www.eso.org/public/news/eso0846/) and (https://www.youtube.com/watch?v=duoHtJpo4GY).

Section 1: First Law

1st Law: "The orbit of every planet is an ellipse with the Sun at one of the two foci."

Ellipses are ovals with a long axis (the "major axis") and a short axis (the "minor axis"). More technically, an ellipse is the set of points where the sum of the distances to each of the foci (r_1 and r_2 in Figure 6.1) is always the same. Sometimes we refer to the semi-major axis, which is just half of the major axis.

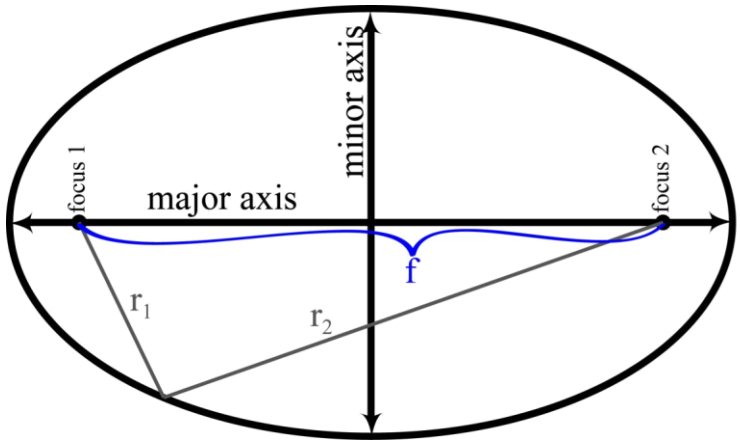

Fig 6.1

The separation of the foci (*f* in Figure 6.1) determines how oval- shaped or "squashed" the ellipse will be: if they are far apart, the major axis will be much larger than the minor axis; but if they are close together, the major and minor axes are almost the same. If the foci are right on top of each other, the ellipse is a circle.

The term "eccentricity" refers to how oval-shaped an ellipse is. Eccentricity ranges from 0 for circular orbits to 1 for an orbit so oval-shaped that it is actually just a line. The larger the eccentricity the more oval-shaped (and less circular) the ellipse.

The eccentricity, *e*, separation between foci, *f*, and the semi-major axis, *a*, are related to each other,

$$e = \frac{f}{2a}$$

How do you draw an ellipse? All you need is a piece of string, two thumbtacks, some cardboard, and a pencil. As shown in Figure 6.2, the thumbtacks should be placed at the foci. Put the cardboard under the paper, so the thumbtacks stay in place. The pencil pulls the string tight and traces out the ellipse.

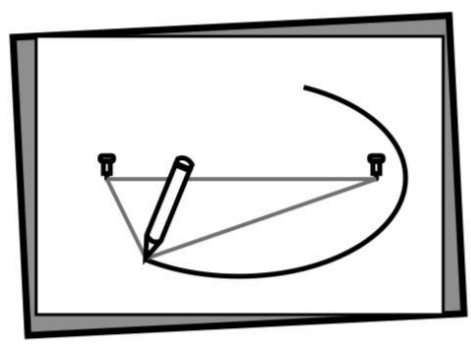

Fig 6.2

How much string do you use? The string length, *s*, is related to the semi-major axis, *a*, and the eccentricity, *e*, by the following:

$$s = 2\,a(1 + e)$$

Remember that at any point along an ellipse, the sum of the distances to the foci remains the same. You see this physically as you draw the ellipse, for the total length of the string stays constant.

In order to get a feeling for how the semi-major axis of an ellipse works, you are going to draw two ellipses with e = 0.2 and a = 6cm, a = 11cm. First, though, you need to do a few calculations that will help your drawing.

Q1.1: What separation between foci should you use for each ellipse? **(2 pts) [T]**

Q1.2: What string length should you use for each ellipse? **(2 pts) [T]**

Your teacher will provide string that you can cut and tape into a loop. On the last page of the lab there are sheets you can use to draw your ellipses.

Q1.3: Use one sheet of paper to draw the two ellipses with f_1 as one of the foci. **(2 pts) [C]**

Q1.4: Describe how increasing the semi-major axis changes the shape of the ellipse. **(1 pt) [C]**

You've worked with ellipses with constant eccentricity and different semi-major axes. To get a feeling for how the eccentricity works, you will draw two ellipses with a = 6cm and e = 0.2, e = 0.75:

Q1.5: What separation between foci should you use for each ellipse? **(2 pts) [T]**

Q1.6: What string length should you use for each ellipse? **(2 pts) [T]**

Q1.7: Use one sheet of paper to draw the two ellipses using f_2 as one of the foci. **(2 pts) [C]**

Q1.8: Describe how increasing the eccentricity changes the shape of the ellipse. **(1 pt)[C]**

Q1.9: Does changing the eccentricity change the semi-major axis? **(1 pt) [C]**

Section 2: Second Law

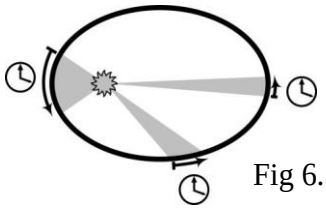

Fig 6.3

2nd Law: "A line joining a planet and the Sun sweeps out equal areas during equal intervals of time."

What does that mean? Why would it take a planet the same amount of time to cover the different parts of its orbit shown in Figure 6.3?

Firstly, let's cover some terminology for planetary orbits. In our solar system the Sun sits at one focus while nothing sits at the other. Astronomers use certain terms to refer to a planetary orbit, shown in Figure 6.4. The size of the orbit is described by the semi-major axis, which is also the average distance between the planet and the Sun. The eccentricity describes the shape of the orbit.

There are two special places in a planet orbit:

- perihelion ("peri-heeleon"), when the planet is closest to the Sun
- aphelion ("ap-heeleon"), when the planet is furthest from the Sun

To help you remember these terms, keep in mind "a" like "away" for aphelion, the farthest distance.

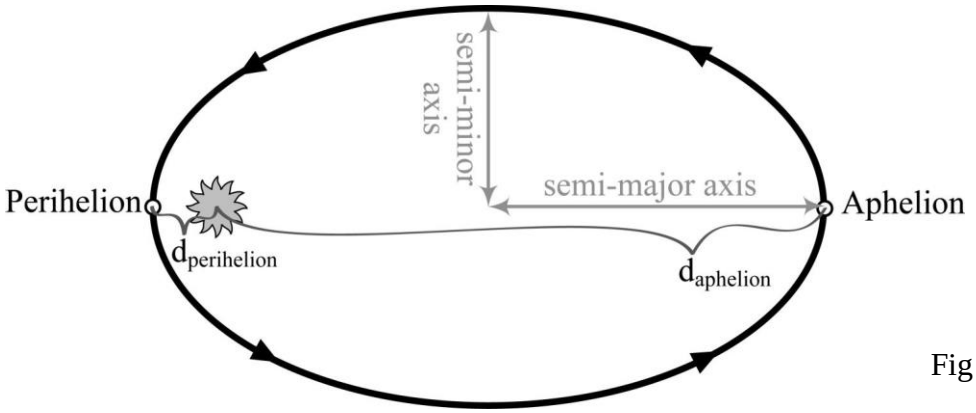

Fig 6.4

To calculate the perihelion and aphelion distances, you use the following:

$$d_{aphelion} = (1 + e)a$$
$$d_{perihelion} = (1 - e)a$$

Just like you would measure the distance between cities in kilometers rather than centimeters, there is a typical unit for measuring planetary orbits: the Astronomical Units (AU). Astronomers define 1 AU as the average distance between the Earth and the Sun and equals 93 million miles or 150 million kilometers. It is far easier to give planetary orbit sizes in AU than in kilometers!

Consider a mythical planet orbiting our Sun with a = 4 AU and e = 0.5.

Q2.1: Given the semi-major axis, is this planet's orbit larger or smaller than Earth's orbit? **(1 pt) [T]**

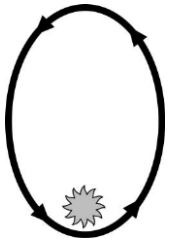

Fig 6.5

Q2.2: What is the aphelion distance? **(2 pts) [T]**

Q2.3: What is the perihelion distance? **(2 pts) [T]**

The phrase "sweeping equal area in equal time" means planet must be moving faster when it is closer to the Sun and slower when it is farther from the Sun as shown in Figure 6.6. What causes this changing distance and velocity? As Newton found out, this is all caused by gravity. The planets are attracted to the Sun due its large mass. However, since the planets have momentum (meaning that they're moving and not standing still) they won't fall directly into the Sun. The result is a dance between gravity and momentum that we call an orbit.

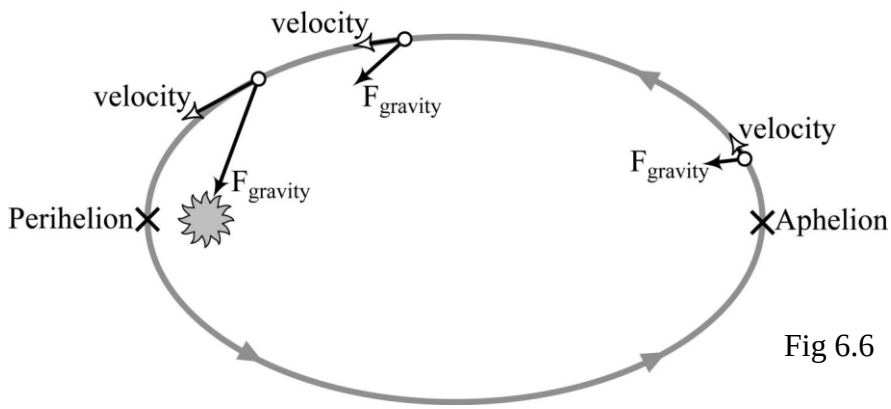

Fig 6.6

The force of gravity on a planet from a star can be calculated if you know the mass, *M*, of the star, the distance between the star and the planet, *d*, and the gravitational constant, *G*:

$$F_{gravity} = \frac{GM_1M_2}{d^2}$$

For planets orbiting our sun the form of the equation you should use to calculate force in this lab is the following.

$$F_{gravity} = \frac{1.18x10^{28}}{d^2}$$

Remember that planet orbiting our Sun with a = 4 AU and e = 0.5? To calculate the force of gravity on the planet you are going to need the perihelion and aphelion distances you calculated in Q2.2 and Q2.3.

Q2.4: Calculate the force at aphelion. **(2 pts) [T]**

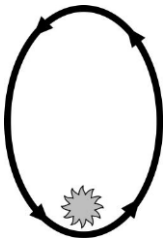

Fig 6.7

Q2.5: Calculate the force at perihelion. **(2 pts) [T]**

Q2.6: At which point is the force greater? **(1 pt) [T]**

Q2.7: Think about the velocity of the planet: if you had to guess, will it be greater at perihelion or aphelion? Explain your reasoning. **(1 pt) [A]**

The velocity, *v*, of a planet at any point in its orbit can be calculated using the *vis viva* equation if you have the semi-major axis, *a*, and the distance from the star at whatever point of interest, *d*:

$$v = \sqrt{GM\left(\frac{2}{d} - \frac{1}{a}\right)}$$

For planets orbiting our Sun the form the equation you should use to calculate velocity in this lab is the following:

$$v = \sqrt{887\left(\frac{2}{d} - \frac{1}{a}\right)}$$

Calculate the orbital velocity of the planet at aphelion and perihelion, using the distances you calculated in Q2.2 and Q2.3.

Q2.8: What is the aphelion velocity? **(2 pts) [T]**

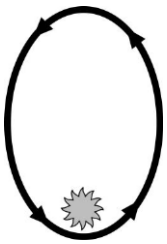

Fig 6.8

Q2.9: What is the perihelion velocity? **(2 pts) [T]**

Q2.10: At which point is the velocity the greatest, perihelion or aphelion? Was your prediction in Q2.7 correct? Explain why or why not. **(1 pt) [A]**

You've worked with an eccentric orbit, **now let's consider a circular orbit of the same size**. A circle is just a special kind of ellipse where the foci are on top of each other. Consider a planet in a circular orbit with a semi-major axis of 4AU, the same semi-major axis as the elliptical orbit you used in Q2.8-2.10. Both orbits are shown in Figure 6.9 for reference but for Q2.11, 2.12, and 2.13 only consider the circular orbit.

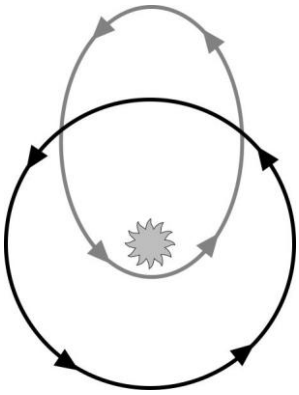

Fig 6.9

Q2.11: For the planet in the circular orbit, how far is it from the star? Does this distance ever change at any point in the orbit? **(2 pts) [C]**

Q2.13: Calculate the force of gravity between the planet and the star. **(2 pts) [T]**

Q2.14: Calculate the planet's orbital velocity. **(2 pts) [T]**

Q2.15: How does the distance for the planet with the circular orbit compare with the distances you calculated for the eccentric orbit in Q2.2 and Q2.3? Which is larger or smaller? Which one is constant throughout the orbit and which one varies? **(1 pt)[A]**

Q2.16: How does the force of gravity on the planet with the circular orbit compare with the forces you calculated for the eccentric orbit in Q2.4 and Q2.5? Which is larger or smaller? Which one is constant throughout the orbit and which one varies? **(1 pt) [A]**

Q2.17: How does the velocity for the planet with the circular orbit compare with the velocities you calculated for the eccentric orbit in Q2.8 and Q2.9? Which is larger or smaller? Which one is constant throughout the orbit and which one varies? **(1 pt) [A]**

Section 3: Third Law

3rd Law: "The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit."

Consider a planet orbiting farther from the Sun and a planet orbiting closer to the Sun, both in circular orbits.

Q3.1: Which planet feels the stronger force of gravity? **(1 pt) [C]**

Q3.2: Which planet has the higher orbital velocity? **(1 pt) [C]**

Q3.3: Which planet travels a greater distance to complete one orbit? **(1 pt) [C]**

Q3.4: Consider the distance each planet travels in one orbit and its orbital velocity. Which planet takes longer to complete one orbit? **(1 pt) [C]**

Kepler's third law allows you determine orbital size or orbital period for a planet orbiting the Sun. The equation relates the semi-major axis of a planet's orbit, *a* in AU, to how long it will

take it to complete one orbit, *P* in Earth years, **but this version of the equation is only valid for a planet orbiting our Sun**:

$$\boldsymbol{a^3 = P^2}$$

Q3.5: If you could move a planet so that it had a smaller orbit closer to the Sun (smaller a), what would happen to its orbital period (get larger, stay the same, get smaller)? **(1 pt) [T]**

Q3.6: Does eccentricity appear in the equation for Kepler's Third Law? Is the orbital period affected by whether an orbit is eccentric or not? **(1 pt) [T]**

Q3.7: For example, the two planet orbits shown in Figure 6.9 have the same semi-major axis of 4AU. How long does it take the circular-orbit planet to complete one orbit? How long does it take the eccentric-orbit planet to complete one orbit? **(1 pt) [A]**

If you are given semi-major axis and need to calculate orbit period, you multiply a by itself three times and then take a square root to get rid of that pesky square. What happens when you are given orbital period and need to determine semi-major axis? You can multiply the period by itself, but how do you get rid of the cube power? A square root is just like taking something to the 1/2 power; so to get rid of the cube, you take it to the 1/3 power.

Q3.8: Use Kepler's third law to complete the first two columns of following table. **(7 pts) [A]**

Planet	**Semi-major Axis (AU)**	**Orbital Period (Earth years)**	**Your age in other planets' years**
Mercury	0.387		
Venus		0.615	
Earth		1	
Mars	1.52		
Jupiter		11.9	
Saturn	9.58		
Uranus	19.2		
Neptune		165	

What about the third column? We measure our age by how many orbits of the Earth we live through, but consider expressing that age using the units of another planet's orbit. If you talked to someone from Mercury and wanted to give your age in "Mercury years", you would have to account the different length of their years (orbital periods):

$$16 \text{ Earth years} \times \frac{1 \text{ Mercury year}}{0.241 \text{ Earth years}} = 66.4 \sim 66 \text{ Mercury years}$$

$$16 \text{ Earth years} \times \frac{1 \text{ Venus year}}{0.615 \text{ Earth years}} = 26.0 \sim 26 \text{ Venus years}$$

Q3.9: Complete the last column by calculating how old you are in other planets' years. **(3 pts) [A]**

Section 4: Review Questions

Q4.1: Consider two planets orbiting a star with the same semi-major axis: one in a circular orbit and one in an elliptical orbit. Draw a sketch of the two orbits around the same star. Think about each planet completing one orbit. Which planet has the maximum orbital speed? Which one has the minimum orbital speed? Which planet experiences a changing force of gravity over its orbit? Which planet does not? Which planet completes its orbit the fastest --- or do they both complete the orbit in the same time? **(3 pts) [A]**

Q4.2: Comets have very elliptical orbits that reach close to the Sun and far out to the edges of the solar system. Comet Hale-Bopp has a semi-major axis of 186 AU. Its orbit is quite eccentric (e = 0.995), so its perihelion distance is 0.914 AU while its aphelion distance is 371 AU. Given what you know about orbital speeds in eccentric orbits, in what part of its orbit does Hale-Bopp spend most of its time? Explain your reasoning. **(3 pts)[A]**

Summary Questions (35 pts)

Answer the following questions in complete sentences, using correct grammar and spelling. Your summary should be about 1 page long, double spaced, 12 point font, without weird (like 1.5") margins.

- What is an ellipse and how do you draw one?
- How does changing the semi-major axis alter an orbit?
- Describe the difference in orbital periods between planets close to the Sun and planets farther away.
- How does changing the eccentricity alter an orbit?
- Summarize your results regarding the orbital velocity and distance to the Sun for when you compared an elliptical orbit and a circular orbit. What were the differences and similarities?
- What keeps a planet orbiting the Sun (hint, what force of nature)?

Background Reading

Throughout human history, the motion of the planets in the sky was a mystery: why did some planets move quickly across the sky, while other planets moved very slowly? Even two thousand years ago it was apparent that the motion of the planets was very complex. For example, Mercury and Venus never strayed very far from the Sun, while the Sun, the Moon, Mars, Jupiter and Saturn generally moved from the west to the east against the background stars (at this point in history, both the Moon and the Sun were considered "planets").

The Sun appeared to take one year to go around the Earth, while the Moon only took about 30 days. The other planets moved much more slowly. In addition to this rather slow movement against the background stars was, of course, the daily rising and setting of these objects. How could all of these motions occur? Because these objects were important to the cultures of the time, even foretelling the future using astrology, being able to predict their motion was considered vital.

The ancient Greeks had developed a model for the Universe in which all of the planets and the stars were each embedded in perfect crystalline spheres that revolved around the Earth at uniform, but slightly different speeds. This is the "geocentric", or Earth-centered model. But this model did not work very well–the speed of the planet across the sky changed. Sometimes, a planet even moved backwards!

It was left to the Egyptian astronomer Ptolemy (85 − 165 AD) to develop a model for the motion of the planets (you can read more about the details of the Ptolemaic model in your textbook). Ptolemy developed a complicated system to explain the motion of the planets, including "epicycles" and "equants", that in the end worked so well, that no other models for the motions of the planets were considered for 1500 years! While Ptolemy's model worked well, the philosophers of the time did not like this model–their Universe was perfect, and Ptolemy's model suggested that the planets moved in peculiar, imperfect ways.

In the 1540's Nicholas Copernicus (1473 − 1543) published his work suggesting that it was much easier to explain the complicated motion of the planets if the Earth revolved around the Sun, and that the orbits of the planets were circular. While Copernicus was not the first person to suggest this idea, the timing of his publication coincided with attempts to revise the calendar and to fix a large number of errors in Ptolemy's model that had shown up over the 1500 years since the model was first introduced. But the "heliocentric" (Sun-centered) model of Copernicus was slow to win acceptance, since it did not work as well as the geocentric model of Ptolemy.

Johannes Kepler (1571 − 1630) was the first person to truly understand how the planets in our solar system moved. Using the highly precise observations by Tycho Brahe (1546 − 1601) of the motions of the planets against the background stars, Kepler was able to formulate three laws that described how the planets moved. With these laws, he was able to predict the future motion of these planets to a higher precision than was previously possible. Many credit Kepler with the origin of modern physics, as his discoveries were what led Isaac Newton (1643 − 1727) to formulate the law of gravity. Today we will investigate Kepler's laws and the law of gravity.

Gravity is the fundamental force governing the motions of astronomical objects. No other force is as strong over as great a distance. Gravity influences your everyday life (ever drop a glass?), and keeps the planets, moons, and satellites orbiting smoothly. Gravity affects everything in the Universe including the largest structures like super clusters of galaxies down to the smallest

atoms and molecules. Experimenting with gravity is difficult to do. You can't just go around in space making extremely massive objects and throwing them together from great distances.

You can model a variety of interesting systems very easily using a computer. By using a computer to model the interactions of massive objects like planets, stars and galaxies, we can study what would happen in just about any situation. All we have to know are the equations that predict the gravitational interactions of the objects. The orbits of the planets are governed by a single equation formulated by Newton:

$$F_{gravity} = \frac{GM_1M_2}{R^2}$$

This law is called an Inverse Square Law because the distance between the objects is squared, and is in the denominator of the fraction. There are several laws like this in physics and astronomy.

The most important thing about gravity is that the force depends only on the **masses of the two objects** and the **distance between them**. A diagram detailing the quantities in this equation is shown in Figure 6.4. Here $F_{gravity}$ is the gravitational attractive force between two objects whose masses are M_1 and M_2. The distance between the two objects is R. The gravitational constant G is just a small number that scales the size of the force.

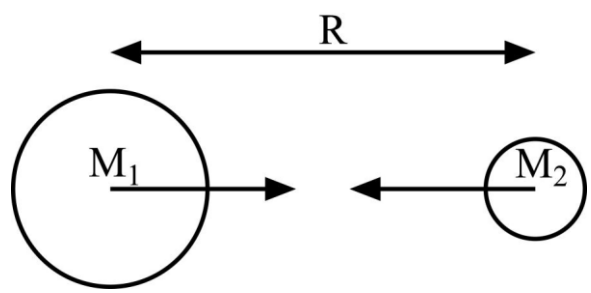

Fig 6.4


f_1


f_2